Abstract

Wild’s summation formula gives an explicit expression for the solution of the spatially homogeneous Boltzmann equation for Maxwellian molecules in terms of its initial data $F$ as a sum $f(v, t) = \sum_{n=0}^{\infty} e^{-t}(1 - e^{-t})^n Q_n^+(F)(v)$. Here, $Q_n^+(F)$ is an average over $n$-fold iterated Wild convolutions of $F$. If $M$ denotes the Maxwellian equilibrium corresponding to $F$, then it is of interest to determine bounds on the rate at which $Q_n^+(F)$ tends to $M$ in $L^1$. In the case of the Kac model, we prove that for every $\epsilon > 0$, if $F$ has moments of every order and finite Fisher information, this convergence is governed by the least negative eigenvalue for the linearized collision operator. We prove that there is a decomposition of $Q_n^+(F)$ into a smooth part and a part that is small for large $n$. This depends in an essential way on a probabilistic construction of McKean. It allows us to circumvent difficulties stemming from the fact that the evolution does not improve the qualitative regularity of the initial data.